RAMAKRISHNA MISSION VIDYAMANDIRA

(A Residential Autonomous College under University of Calcutta)

First Year

First-Semester Examination, December 2010

Date : 15-12-2010 **COMPUTER SCIENCE (Honours)** Full Marks: 75

Time : 11am – 2pm **Paper - I**

Answer <u>any five</u> questions from the following: [5×15 = 75]

1. a) Answer <u>any five.</u>
   i) Let A = {1, 2, …, n} where n>10 and B = {w, x, y, z}. What is the number of one-to-one functions from B to A? [1]
   ii) Choose the right answer from the alternatives. A function $f : R \to R$ is defined by $f(x) = e^{-x}$. Then f(x) is—
   a) injective b) surjective c) bijective d) not a function [1]
   iii) Let A, B be 2 sets and let $R \subseteq A \times B$ such that $\forall a \, \forall b ((a,b) \in R \Rightarrow (b,a) \in R)$. What kind of a relation is R? [1]
   iv) What is the time complexity of Strasser's algorithm for multiplying two n×n matrices of integers? [1]
   v) Let G be a graph with n vertices and m edges. What is the time complexity of depth first traversal of G? [1]
   vi) If a graph G has no odd cycles, what can one say about G? [1]

   b) i) What is the minimum and maximum height of a binary tree containing n nodes? [2]
   ii) For a certain base x, the equation $\sqrt{(41)_x} = (5)_x$ is valid. What is the value of x? [2]
   iii) What is the smallest value of k for which $f(n) = n\log(n^2+1) + n^2\log n$ is $0(n^k)$? [2]
   iv) Convert the number $(0{\cdot}625)_{10}$ to its octal form. [2]
   v) Match the following : [2]

Divide and Conquer	Binary search
Dynamic programming	Huffman coding
Greedy algorithm	Fibonacci sequence

2. a) Define a regular graph. [1]
   b) Prove that every tree contains at least two vertices of degree one. [4]
   c) i) Define the chromatic number of a graph. [2]
   ii) What is a complete bipartite graph? [1]
   iii) What is the chromatic number of
   $K_{4,10}$ [1]
   $K_{5,?}$ [1]
   Justify your answer.
   d) Consider a simple graph with n vertices and k components. Prove that the number of edges in it can not be more than [(n – k)(n – k + 1) / 2]. [5]

3. a) Recall that a pack of cards contains 13 cards in each of 4 suits. What is the minimum number of cards that one must draw in order to ensure that at least 3 cards belong to the same suit? [2]
   b) Let S = {1, 2, …, 2n}(n>1). In how many ways can one pick 3 distinct numbers from S such that these numbers are in arithmetic progression? [4]

c) Define a recurrence relation for Fibonacci sequence. Use it to find its generating function. [4]

d) Use Warshall's algorithm to find transitive closure of the relation R = {(1,2), (2, 3), (3,4), (5,3), (2,2)} defined on A = {1,2,3,4,5}. [5]

4. a) Write pseudo-code for the mergesort algorithm. What is the time complexity of your algorithm? [5]

b) Let A, B, C and D be 10×20. 20×100, 100×50 and 50×1000 matrices of integers. Calculate the minimum number of integer multiplications required to compute ABCD. [5]

b) Illustrate with a non-trivial example, how you can use Kruskal's algorithm to find a maximal spanning tree of a connected weighted graph. [5]

5. a) Assuming that 1 (High) and 0 (low) lines are available in addition to the input lines, implement a 2-input Ex-NOR gate using two 2-input Ex-OR gates. [3]

b) Design a sequential circuit using JK flipflop to implement the following state equations:

$A(t + 1) = xAB + yA'C + xy$

$B(t + 1) = xAC + y'BC'$

$C(t + 1) = x'B + yAB'$ [6]

d) A 4-input Boolean function is given by : $f(a,b,c,d) = \sum(0,1,2,3,4,6,7,8,9,11,15)$ Minimize it using Karnaugh map. [6]

6. a) Design a 2's complemented adder/substractor using 4-bit Full adder (IC 7483) and four 2-input XOR gates with a control line named add/sub. [4]

b) Explain the working principle of Digital-to-Analog data converter with help of R-2R ladder method. [4]

c) A logic circuit is to be designed to carry out modulo-4 addition. The circuit takes two 2 bit inputs (x_1x_2 and y_1y_2) and produces a 2-bit output (z_1z_2). The value of z_1z_2 is 00, 01, 10 or 11 depending on whether the sum of x_1x_2 and y_1y_2 is 0, 1, 2 or 3 modulo 4. Find the Boolean expressions for z_1 and z_2 in terms of x_1, x_2, y_1 and y_2. Minimize these expressions if possible. [7]

7. a) Illustrate the steps for Binomial Coefficient calculation using dynamic programming to compute ${}^5C_4$. [5]

b) Write Dijkstra's algorithm for finding the shortest cost between two vertices in a connected graph with positive edge weights. [5]

c) Assume that you want to send a message of length 1000 consisting of only five symbols, i.e. 'a', 'b', 'c', 'd' and 'e'. You observe that in the message 'a' occurs for 200 times, 'b' for 350 times, 'c' for 250 times, 'd' for 100 times and the symbol 'e' occurs for 100 times. Following the Huffman encoding technique, assign suitable codes to the symbols. [5]